Unit: Rotation

I. Coordinates: \( \theta = \frac{s}{R}, \omega = \frac{v_t}{R}, \alpha = \frac{a_t}{R} \)

II. Kinematics: *same form, new variables*

III. Energy:
   A. Moment of Inertia (rotational mass): \( I = \sum mr^2 \)
   B. Rotational Kinetic Energy: \( K = \frac{1}{2}I \omega^2 \)
   C. Rolling Bodies: *only the type of shape matters*

IV. Torque (rotational force): \( \tau = rF\sin\theta \)

V. Angular Momentum:
   A. \( L = rpsin\theta = I\omega \)
   B. \( \sum \tau \cdot t = \Delta L \)
   C. Conservation
Rotational coordinates are:

\[ \theta = \frac{s}{r} \quad \omega = \frac{v_t}{r} \quad \alpha = \frac{a_t}{r} \]

Therefore:

**OLD EQUATIONS**

\[ v_f = v_i + at \]
\[ x_f = x_i + v_i t + 0.5at^2 \]
\[ x_f = x_i + 0.5(v_i + v_f)t \]
\[ v_f^2 = v_i^2 + 2a(x_f - x_i) \]

**NEW EQUATIONS**

\[ \omega_f = \omega_i + \alpha t \]
\[ \theta_f = \theta_i + \omega_i t + 0.5\alpha t^2 \]
\[ \theta_f = \theta_i + 0.5(\omega_i + \omega_f)t \]
\[ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \]
A vehicle is traveling counterclockwise in a circle (r=40m) at a constant speed of 10m/s. At the moment shown below, what is the vehicle’s angular velocity?
A vehicle is traveling counterclockwise in a circle (r=40m) at a constant speed of 10m/s. At the moment shown below, what is the vehicle’s angular acceleration?
A vehicle is traveling counterclockwise in a circle (r=40m) at a constant speed of 10m/s. At the moment shown below, the driver hits the brakes. If the car stops after \( \frac{1}{4} \) revolutions around the track, what angular acceleration did the brakes provide?
A vehicle is traveling counterclockwise in a circle $(r=40\text{m})$ at a constant speed of $10\text{m/s}$. At the moment shown below, the driver hits the brakes. If the car stops after $\frac{1}{4}$ revolutions around the track, what tangential acceleration did the brakes provide?
A top spins around 10 times every second. There is a red mark on the top 5cm from the center. How fast is the mark moving?
A top spins around 10 times every second. If the top stops after 1 minute, what is its angular acceleration?
A top spins around 10 times every second. There is a red mark on the top 5cm from the center. If the top stops after 1 minute, what is the tangential acceleration of the red mark?
You are in the 8m-diameter spinning fair ride below, traveling at 2m/s. The controller speeds up the ride at a rate of 2m/s\(^2\) (counterclockwise). What is the magnitude of your angular acceleration?
You are in the 8m-diameter spinning fair ride below, traveling at 2m/s. The controller speeds up the ride at a rate of 2m/s\(^2\) (counterclockwise). Determine your angular displacement during the first 3 seconds after the controller began speeding up the ride.
You are in the 8m-diameter spinning fair ride below, traveling at 2m/s. The controller speeds up the ride at a rate of 2m/s\(^2\) (counterclockwise). Determine your angular speed 3 seconds after the controller began speeding up the ride.
A disk hangs from a string as shown. When the system is wound, the string acts as a torsion spring, causing the disk to start spinning in the opposite direction (such as to unwind the string).
The disk is initially spinning counterclockwise at 2 rev/s. 8 seconds later, it is spinning clockwise at 3 rev/s. What was the average angular acceleration?
A disk hangs from a string as shown. When the system is wound, the string acts as a torsion spring, causing the disk to start spinning in the opposite direction (such as to unwind the string).

The disk is initially spinning counterclockwise at 2 rev/s. 8 seconds later, it is spinning clockwise at 3 rev/s. What was the total angular displacement during the 8 seconds?
A 1,000kg vehicle is traveling counterclockwise in a circle (r=40m) at a constant speed of 10m/s. What is the vehicle’s moment of inertia?
Three more cars are added and the radius of the track is doubled. What happens to the total moment of inertia?
Two objects of equal mass rotate at the same rate. If object A is solid and object B is a shell. Which has the most rotational kinetic energy?
The rod below has negligible mass. The blue balls are each 2kg. The green ball is 1kg. What is the moment of inertia of the system when it rotates about the center? (Ignore the size of the balls.)
The rod below is 2kg. The blue balls are each 2kg. The green ball is 1kg. What is the moment of inertia of the system when it rotates about the center? (Ignore the size of the balls.)
The rod below is 2kg. The blue balls are each 2kg. The green ball is 1 kg. What is the rotational kinetic energy of the system when it rotates 0.5 times per second about the center? (Ignore the size of the balls.)
A top \((r=0.5\text{m})\) spins around 10 times every second. If most of the mass is concentrated in the disk, what is the rotational kinetic energy of the top?

\[
I_{\text{disk}} = \frac{1}{2}MR^2
\]
A top \((r=0.5\text{m})\) spins around 10 times every second. If most of the mass is concentrated in the disk, what is the rotational kinetic energy of the top?
A 4kg ball rotates around the green post below 3 times per minute. The ball has a radius of 2m and the center of the ball is 5m away from the post. What is the rotational kinetic energy of the ball.
A 9kg solid cylinder rotates around its center. The cylinder has a radius of 4m, but it has a sphere cut out of one side (r = 2m). The volume of the sphere accounts for 10% of the total volume of the cylinder. What is the moment of inertia of the cylinder? (\( I_{\text{cylinder}} = \frac{1}{2}MR^2 \), \( I_{\text{sphere}} = \frac{2}{5}MR^2 \))