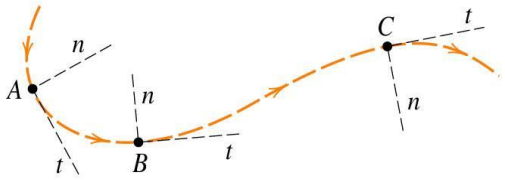
COORDINATE SYSTEMS

I. NORMAL AND TANGENTIAL COORDINATES



A. Velocity $\vec{v} = v\hat{t}$

B. Acceleration $\vec{a} = \dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}$

II. RECTANGULAR COORDINATES

A. Position $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

B. Velocity $\vec{v} = \dot{\vec{r}} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$

C. Acceleration $\vec{a} = \dot{\vec{v}} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$

III. POLAR COORDINATES

A. Coordinates

$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2}$$

$$y = r\sin\theta \qquad \theta = \tan^{-1}\left(y/x\right)$$

B. Unit Vectors

$$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

$$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

$$\hat{\imath} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

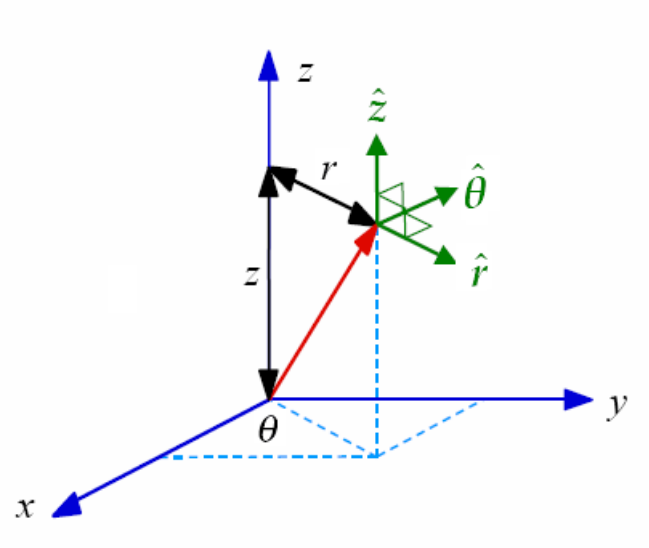
$$\hat{\jmath} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$

C. Position Vector $\vec{r} = r\hat{r}$

D. Velocity $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

E. Acceleration $\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{\theta}$

IV. CYLINDRICAL COORDINATES (3D)



A. Coordinates

$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2}$$

$$y = r\sin\theta \qquad \theta = \tan^{-1}\left({}^{y}/_{x}\right)$$

$$z = z \qquad z = z$$

B. Unit Vectors

$$\hat{r} = \cos\theta\,\hat{r} + \sin\theta\,\hat{j}\hat{\imath} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{j}\hat{j} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$

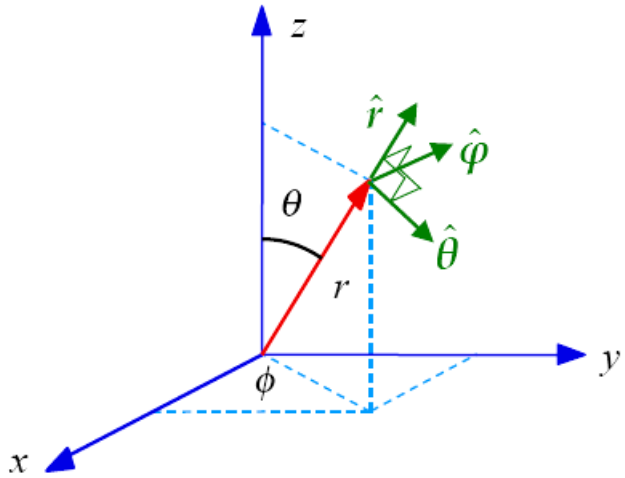
$$\hat{k} = \hat{k}\hat{k} = \hat{k}$$

C. Position Vector $\vec{r} = r\hat{r} + z\hat{k}$

D. Velocity $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{k}$

E. Acceleration $\vec{a} = \dot{\vec{v}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{\theta} + \ddot{z}\hat{k}$

V. SPHERICAL COORDINATES



A. Coordinates

$$x = r\,\boldsymbol{sin}\,\theta\,\boldsymbol{cos}\,\varphi \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$y = r\,\boldsymbol{sin}\,\theta\,\boldsymbol{sin}\,\varphi \qquad \varphi = \boldsymbol{tan}^{-1}\left({}^{y}/_{x}\right)$$

$$z = r\,\boldsymbol{cos}\,\theta \qquad \theta = \boldsymbol{cos}^{-1}\left({}^{z}/_{r}\right) = \boldsymbol{cos}^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

B. Unit Vectors

$$\hat{r} = \boldsymbol{sin}\,\theta\,\boldsymbol{cos}\,\varphi\,\hat{r} + \boldsymbol{sin}\,\theta\,\boldsymbol{sin}\,\varphi\,\hat{\jmath} + \boldsymbol{cos}\,\theta\,\hat{k}$$

$$\hat{\theta} = \boldsymbol{cos}\,\theta\,\boldsymbol{cos}\,\varphi\,\hat{\imath} + \boldsymbol{cos}\,\theta\,\boldsymbol{sin}\,\varphi\,\hat{\jmath} - \boldsymbol{sin}\,\theta\,\hat{k}$$

$$\hat{\varphi} = -\,\boldsymbol{sin}\,\varphi\,\hat{r} + \boldsymbol{cos}\,\varphi\,\hat{\jmath}$$

$$\hat{\imath} = \boldsymbol{sin}\,\theta\,\boldsymbol{cos}\,\varphi\,\hat{r} + \boldsymbol{cos}\,\theta\,\boldsymbol{cos}\,\varphi\,\hat{\theta} - \boldsymbol{sin}\,\varphi\,\hat{\varphi}$$

$$\hat{\jmath} = \boldsymbol{sin}\,\theta\,\boldsymbol{sin}\,\varphi\,\hat{r} + \boldsymbol{cos}\,\theta\,\boldsymbol{sin}\,\varphi\,\hat{\theta} + \boldsymbol{cos}\,\varphi\,\hat{\varphi}$$

$$\hat{k} = \boldsymbol{cos}\,\theta\,\hat{r} - \boldsymbol{sin}\,\theta\,\hat{\theta}$$

C. Position Vector $\vec{r} = r\hat{r}$

D. Velocity $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\varphi}\,\boldsymbol{sin}\,\theta\,\hat{\varphi}$

E. Acceleration

$$\vec{a} = \left(\ddot{r} - r\dot{\varphi}^2\,\boldsymbol{sin}^2\,\theta - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\,\boldsymbol{sin}\,\theta\,\boldsymbol{cos}\,\theta\right)\hat{\theta}$$
$$+ \left(r\ddot{\varphi}\,\boldsymbol{sin}\,\theta + 2\dot{r}\dot{\varphi}\,\boldsymbol{sin}\,\theta + 2r\dot{\theta}\dot{\varphi}\,\boldsymbol{cos}\,\theta\right)\hat{\varphi}$$