Guidelines for the method of undetermined coefficients

Given the *constant coefficient* linear differential equation

$$ay'' + by' + cy = g(t),$$

where $g(t)$ is an exponential, a simple sinusoidal function, a polynomial, or a product of these functions:

1. Solve the homogeneous equation for a pair of linearly independent solutions $y_1(t)$ and $y_2(t)$.
2. If $g(t)$ is *not* a solution of the homogeneous equation, take a trial solution of the same type as $g(t)$ according to the suggestions given in Table 2.1.
3. If $g(t)$ is a solution of the homogeneous equation, take a trial solution of the same type as $g(t)$ multiplied by the lowest power of t for which no term of the trial solution is a solution of the homogeneous equation.
4. Substitute the trial solution into the differential equation and solve for the undetermined coefficients so that it is a particular solution $y_p(t)$.
5. Set $y(t) = y_p(t) + c_1y_1(t) + c_2y_2(t)$, where the constants c_1 and c_2 can be determined if initial conditions are given.
6. If $g(t)$ is a sum of the type of forcing functions described above, split the problem into simpler parts. Find a particular solution for each of these, then add the particular solutions to obtain $y_p(t)$.

$g(t)$	Trial solution
ae^{rt}	Ae^{rt}
$a \sin \omega t$ or $a \cos \omega t$	$A \sin \omega t + B \cos \omega t$
at^n, where n is a positive integer	$P(t)$, a general polynomial of degree n
at^ne^{rt}, where n is a positive integer	$P(t)e^{rt}$, with $P(t)$ a general polynomial of degree n
$t^n(a \sin \omega t + b \cos \omega t)$, where n is a positive integer	$P(t)(A \sin \omega t + B \cos \omega t)$, with $P(t)$ a general polynomial of degree n
$e^{rt}(a \sin \omega t + b \cos \omega t)$	$e^{rt}(A \sin \omega t + B \cos \omega t)$